Mth 112 — Tuesday / Weds 11-13 / 11-14 — Class Notes

Ch 11-4 The Common Logarithms

Note: the "Antilog" is just the inverse of the common log function ⇒ read "the number that has a log of..." (Exponent)

Ex: $\text{Antilog } x = N$

"the number that has an exponent of x is N"

LOOK ⇒ * BASE IS 10

$\therefore \; 10^x = N$

Ex: $\text{Antilog } 2 = ?$ $\quad 10^2 = \boxed{100}$

Ex: $\text{Antilog } -3.4 = ?$ $\quad 10^{-3.4} = \boxed{3.98 \times 10^{-4}} = .000398\ldots$

Ch. 11-5 Logarithms To Base e

Natural Logarithms $\Rightarrow$ ln

$$\log_e X = \ln X$$

Many "natural" sources of e

One source (i.e., where you find it when you model a natural process mathematically)

$\Downarrow$

Continuous Growth or Decay

$$A = A_0\left(1 \pm \frac{r}{N}\right)^{Nt}$$

As N approaches "continuous" growth or decay

$\Downarrow$

CONTINUOUS

$\Downarrow$

$$\boxed{A = A_0 e^{\pm rt}}$$

Exercise:
use a spreadsheet to estimate $A = A_0\left(1 \pm \frac{r}{N}\right)^{Nt}$ if A_0, r, t all are 1 and N gets large.

EditGrid

Explo

Spreadsheet / Untitled

File Edit View Format Insert Data Share Publish Collaborate Macro Help

Arial 10 pt

D21 fx

Please save your spreadsheet. (hide)

Spreadsheet Name

	A	B	C	D	E	F
1	r	n	t	rate: r/n	changes: n*t	formula: (1+r/n)^nt
2	1	1	1	1	1	2
3	1	2	1	0.5	2	2.25
4	1	10	1	0.1	10	2.5937424601
5	1	20	1	0.05	20	2.65329770514442
6	1	200	1	0.005	200	2.71151712292932
7	1	1000	1	0.001	1000	2.71692393223559
8	1	10000	1	0.0001	10000	2.71814592682493
9	1	20000	1	5E-05	20000	2.71821387453362
10	1	30000	1	3.33333333333333E-05	30000	2.71823652514587
11	1	40000	1	2.5E-05	40000	2.71824785070851
12	1	50000	1	2E-05	50000	2.71825464612673
13	1	60000	1	1.66666666666667E-05	60000	2.71825917647435
14	1	70000	1	1.42857142857143E-05	70000	2.71826241242026
15	1	80000	1	1.25E-05	80000	2.71826483938595
16	1	90000	1	1.11111111111111E-05	90000	2.71826672706496
17	1	100000	1	1E-05	100000	2.7182682371923
18	1	1000000	1	1E-06	1000000	2.71828046909573
19	1	20000000	1	5E-08	20000000	2.71828175605259
20	1	10000000000	1	1E-10	10000000000	2.71828205269085

$\leftarrow e$

Exercise: Using a free online spreadsheet I found at "EditGrid.com" I estimated what $A = A_0\left(1+\frac{r}{n}\right)^{nt}$ approaches if $A_0 = 1$, $r = 1$, $t = 1$

that is: $1\left(1+\frac{1}{N}\right)^{1N}$

or $\left(1+\frac{1}{N}\right)^{N}$ as N gets very big.

This is why "e" appears as the base for "continuous" change

Some Examples

Ch. 11-4 # 41 $pH = -\log[H^+]$

concentration of hydrogen ions H^+, in moles per liter of water

$1 \text{ mole} \approx 6.02 \times 10^{23}$

Find pH if $[H^+]$ is $5 \times 10^{-4} \frac{\text{moles}}{l}$

$$pH = -\log[5 \times 10^{-4}]$$

$$pH = -[\log 5 + \log 10^{-4}]$$

$$= -[.69897 + -4 \underbrace{\log 10}_{1}]$$

$$= -[.69897 - 4]$$

$$= -[-3.30103]$$

$$\boxed{pH \cong +3.3} \Rightarrow$$

very acidic

$pH > 7 = \text{basic}$
$pH = 7 \Rightarrow \text{neutral}$
$pH < 7 = \text{acidic}$

See online next pg

pH - Mozilla Firefox
File Edit View History Bookmarks Tools Help
www.rpi.edu/dept/chem-eng/Biotech-Environ/EQUILIBRIUM/ph.htm
pH calculator
Bookmarks Math WebMail iGoogle Goo.gl Arduino Social News Shop iNOW EdDir Router Ham Radio Cool SSCC Mars Science Laboratory Holt McDougal Online WoT Wiki
pH

pH

It is inconvenient to express concentration of hydrogen ions as a number with many zeros right after the decimal point, e.g., $H^+ = 0.00000048$ N. Instead we define pH as the negative logarithm of the hydrogen ion concentration, and the clumsy number just used becomes pH = 6.32. Pocket calculators convert concentration to pH merely by entering the concentration, invoking the logarithm (base 10) function, and changing the sign. The reverse operation requires entering the pH, raising ten to that power, and taking the reciprocal.

Enter pH 3.30103 H^+ = 0.0004999999950079738
or enter concentration of hydrogen ions .0005 pH = 3.301
Perform calculation

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BTW: I "googled" "ph calculator" and immediatly found several interactive pH calculators. This simple one from RPI (a great school for science and engineering) is at:
www.rpi.edu/dept/chem-eng/Biotech-Environ/EQUILIBRIUM/ph.htm

Ch 11-5
Ex 11-5C
Pg 506

How long to double world population if: $r = 2\%$ per year, compounded continuously?

Use $P = P_0 e^{+rt}$

To douple $\frac{P}{P_0} = \frac{2}{1}$

$\therefore \quad 2 = e^{(.02)t}$

Take $\underline{\underline{\ln}}$ of both sides

"see e, use e" !! That's why e is in your calculator.

$\ln 2 = \ln e^{.02t}$

$.693147 = .02t \ln e$

By definition $\ln e = 1$

$\frac{.693147}{.02} = t$

$34.657 = t \quad \therefore \boxed{t \approx 34.7 \text{ years}}$

* Note: Same idea for decay; use ratio of start to end values — see Ex 2 Pg 506

(Ex) Using round numbers, the US population is $\approx 300 \times 10^6$ and the world population is $\approx 7 \times 10^9$. How many years to add "one US"?

Enter "world population growth rate" into WolframAlpha and you get

see next pg

$r \cong 1.14\% = .0114$

Using $P = P_0 e^{(.0114)t}$

$$\frac{P}{P_0} = \frac{7\times10^9 + 3\times10^8}{7\times10^9} = \frac{7.3}{7}$$

$$\frac{P}{P_0} = 1.042857143$$

$$\therefore\ 1.042857143 = e^{(.0114)t}$$

$$\ln(1.042857143) = .0114t$$

$$\frac{.041964199}{.0114} = t$$

$$\boxed{3.68\text{yrs} \cong t}$$

WolframAlpha computational.. knowledge engine

world population growth rate

Examples Random

Input interpretation:

world | population growth

Result:

1.14 %/yr (percent per year) (2006 estimate)

Demographics:

Show rates | Show distribution | Show metric

population	6.79 billion people (2009 estimate)
population density	118 people/mi^2 (people per square mile) (2009 estimate)
population growth	1.14 %/yr (2006 estimate)
life expectancy	64.8 years (2009 estimate)
median age	27.6 years

Units »

Computed by Wolfram *Mathematica*

Sources Download page

WolframAlpha computational knowledge engine

world population since 1700 ⇒ "good" numbers start @ ≈ 1800

Examples Random

Input interpretation:

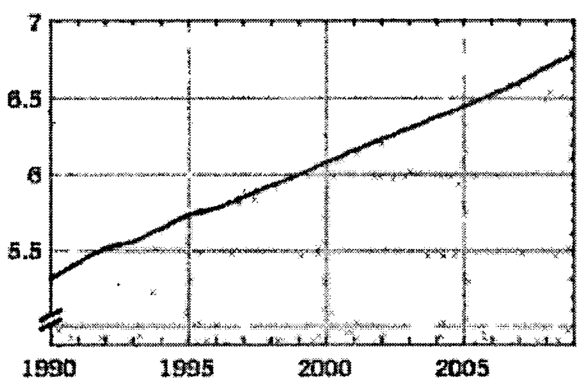
world	population	1700 to today

Result:

mean	2.119 billion people
lowest	959.4 million people (1800)
highest	6.79 billion people (2009)

Recent population history:

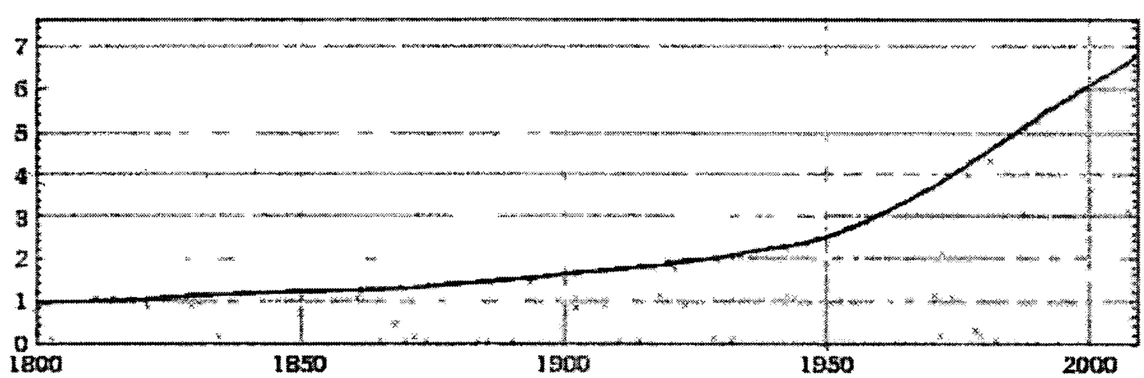
Log scale



(from 1990 to 2009)
(in billions of people)

Long-term population history:

Log scale



(from 1800 to 2009) (in billions of people)

LOOK

Ch. 11-6 Exponential Equations

your variable is "trapped" in an exponent

Two possibilities ⇒ ① you "luck out" and you can make each sides base the same

ACT uses this type

Ex $9^{3b-3} = 27^{2b+2}$

$$(3^2)^{3b-3} = (3^3)^{2b+2}$$

$$3^{6b-6} = 3^{6b+6}$$

$$\therefore \quad 6b-6 = 6b+6$$

$$-6 = 6 \text{ False, } \boxed{\text{No Solution}}$$

Ex $25^{-r} = 125$

$$(5^2)^{-r} = 5^3$$

$$5^{-2r} = 5^3$$

$$\therefore \quad -2r = 3$$

$$\boxed{r = -\tfrac{3}{2}}$$

CK $25^{-\frac{3}{2}} \stackrel{?}{=} 125$

$$\frac{1}{25^{\frac{3}{2}}} \stackrel{?}{=} 125$$

$$\frac{1}{(\sqrt[2]{25})^3} \stackrel{?}{=} 125 \checkmark$$

Second Possibility: you did not "luck out", you cannot get same bases on both sides

$\Rightarrow$ take logarithm of both sides

Tip: you can take $\log_{10}$ or $\log_e$ of both sides but...

($\log_e$ = ln)

* see base 10 use log, see e, use ln

Ex 1 pg 509

$$3^{x+7} = 11$$

$$\log\left[3^{x+7}\right] = \log 11$$

calculator

$$\frac{(x+7)\cancel{\log 3}}{\cancel{\log 3}} = \frac{1.041392685}{0.477121255}$$

$\leftarrow \log 3$

$$x = 2.182658339 - 7$$

$$\boxed{x \cong -4.8173}$$

ck $\quad 3^{(-4.8173+7)} \stackrel{?}{=} 11$

$$11.00050 \stackrel{?}{=} 11 \checkmark$$